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An Analysis of the Consequences of Certain Patterns of Immigration

\. Introduction

THE countries that admit immigrants on a regular basis usually maintain A detailed records of their foreign born population. The size and distribution of the foreign born residents with respect to their demographic and social characteristics relative to those of the native population, are naturally determined by the immigration policies of their Governments. The effects of the different policies on the growth and other characteristics of a population experiencing specified patterns of vital rates are worthy of critical examination. Some of the recent studies in this area include cases in which the net maternity rates remain the same and a constant number of immigrants with unchanging age composition enter every year in a country where the net reproduction rate is (a) less than one (Espenshade, 1982) and *(b)* equal to or greater than one (Mitra, 1983). The invariance of both the overall net migration rate and the age composition of the immigrants have also been examined in the same context (Sivamurlhy, 1982).

In this paper we shall deal with the problem of determining the eventual age composition of the immigrants relative to the native born under two different immigration policies. According to the first, a fixed number of immigrants with unchanging age composition is allowed entrance each year whereas a constant schedule of age-specific immigration rates is maintained in the second. In the first example, the contribution of the immigrants to the age composition of the population relative to that of the native born gets smaller and smaller when the population experiences a non-negative rate of growth which is the case when the net reproduction rate R is greater than or equal to one. It may be pointed out that for R = I, the birth trajectory of the population becomes linear (Mitra, *ibid)* with a positive slope under such a policy of immigration. However, when $R < 1$ and the population eventually becomes stationary in the same example, the solution giving the age composition of the immigrants relative to the native born becomes quite interesting. In the other example, where the age-specific immigration rates remain constant, the solution naturally exists for all values of the net reproduction rate. Specifications of the conditions and the analytical solutions that follow from them have been outlined in the subsequent sections.

2. Unchanging Age-Specific Immigration Rates

For reasons of simplicity let us begin with the definition of a situation in which the probability of survival from birth to any age a or $p(a)$ remains invariant over time together with the age-specific immigration rate $i(a)$. It is intuitively apparent that the continuous operation of this process will result in a constant proportion $n(a)$ that the first generation immigrants will bear to the native born population aged a. What is not apparent is the relationship between $i(a)$ and $n(a)$ towards a derivation of which we turn next. As will be seen we have found it operationally simpler to derive $i(a)$ from $n(a)$ rather than the other way around. Again, to keep things simple, we have also restricted ourselves to only one sex, namely the females. We now begin by writing the female population aged a at time / or $F(a, t)$ as

$$F(a, t) = B(t-a)p(a)[1 + n(a)], \quad a>0 \tag{1}$$

Where $B(t — a)$ is the total number of births at time $t — a$ and as such $B(t — a)\, p(a)$ gives the size of the native born population aged a. Similarly, $B(t — a)\, p(a)\, n(a)$ is the size of the first generation immigrants which follows from the definition of $n(a)$. The functions appearing in (1) are continuous and as $p(a)$ *is* monotonic, nonnegative and nonincreasing with $p(0) = 1$, $n(a)$ by definition is monotonic, non-negative and non-decreasing with $n(0) = 0$. This is so because immigrants have to be born elsewhere and accordingly for them, the age of immigration has to be greater than zero, i.e., $a > 0$.

In order to obtain the number aged a who immigrate at time t or $N(a, t)$, we first subtract from

$$F(a + h, T + h) = B(t - a)\, p(a+h)\, [1+n(a+h)\} \tag{2}$$

the survivors of $F(a, t)$ at time $t + h$, namely,

$$F(a, t)\, \frac{P(a+h)}{P(a)} = B(t-a)\, P(a + h)\, (1 + n(a)] \tag{3}$$

to get

$$B(t - a)\, P(a+h)\, [n(a+h)- n(a)] \tag{4}$$

Here we have made the assumption that like the host population, the immigrants are also subjected to the same mortality conditions. Next, we divide (4) by *HP* and then obtain its limiting value as *h* -> 0 to get the desired number aged *a* who immigrate at time *t.* Thus,

$$N(a, t) = \lim_{h \to 0} B(t - a) P(a + h) \frac{n(a + h) - n(a)}{h}$$

$$= B(t-a)\ P(a)n'(a) \qquad (5)$$

Therefore, like *n(a),* the age-specific immigration rate

$$i(a, t) = \frac{N(a, t)}{F(a, t)} = \frac{n'(a)}{1 + n(a)} = i(a) \qquad (6)$$

is also independent of time. We would like to note that for the derivation of (6) it is not necessary to assume the invariance of *p(a),* but since we need that to simplify some of our later derivations we went ahead with the assumption at the outset. Now, we turn to the solution of the differential equation (6) which expresses *n(a)* in terms of /(a) as

$$\ln[1 + n(a)] = \int_0^a i(x)\, dx + \ln k \qquad (7)$$

where In/c is the constant of integration. Since n(0) = 0, we get *k* = 1 and therefore,

$$\ln [1 + n(a)] = \int_0^a i(x)\, dx \qquad (8)$$

This shows how the continuous operation of a given schedule of age-specific immigration rates can determine the distribution by age of the ratio of foreign born to native population. For the finite approximation of (8) we note that the immigration rate for an age interval *(z, z + h)* or *i(z, z + h)* can be expressed approximately as

$$i(z, z + h) = \frac{1}{h} \int_0^h i(z + x)\, dx \qquad (9)$$

Therefore, we can write (8) as

$$\text{In } [1 + n(a)] = h \mathop{E}_{r=0}^{(a-h)/h} i(rh, (r + 1) h) \qquad (10)$$

where a is a multiple of the age interval *h.*

It may be pointed out in this context that the effect of immigration in the pattern postulated in the foregoing can be looked upon as an adjustment to the mortality rates. That is to say, instead of a survivorship probability $p(a)$, we now have $p(a)\,[1 + n(a)]$ which we shall hereafter denote by $t(a)$ and in the absence of a better term use it in the sense of a proportion of attrition. However, like $p(a)$ the proportion $t(a)$ does not have to be a nonincreasing function of age.

Now that $n(a)$ is known, the size of the immigrant population aged a at time t or $I(a, t)$ can be determined from

$$I(a, t) = B(t - a)\, p(a)\, n(a), \quad t > a \tag{11}$$

Summing over age, the total number of immigrants can then be expressed as

$$I(t) = \int_0^\infty B(t - a)\, p(a)\, n(a)\, da \tag{12}$$

for large t and similarly, the size of the native born citizens $C(t)$ as

$$C(t) = \int_0^\infty B(t - a)\, p(a)\, da \tag{13}$$

The overall proportion of immigrants to native born population can be computed from (12) and (13) as

$$i(t) = I(t)/C(t) \tag{14}$$

Assuming invariance of age-specific birth rates $m(a)$ at age a over time we first write $B(t)$, the number of births at time t as (see eqn. 1)

$$B(t) = G(t) + H(t) + \int_0^t B(t - a)\, t(a)\, m(a)\, da \tag{15}$$

in which $G(t)$ stands for the number of births at time t to the population who were residing in the country prior to the start of the process and $H(t)$ is the number of births to immigrants who are more than t years old. Naturally, $G(t), H(t) = 0$ for $t > \beta$, the upper boundary of the reproductive interval.

We can decompose $B(t)$ into two additive components,

$$B(t) = B_o(t) + B_i(t) \tag{16}$$

where

$$B_o(t) = G(t) + \int_0^t B(t - a)\, p(a)\, m(a)\, da \tag{17}$$

gives the number of births to native born mothers and

$$B_i(t) = H(t) + \int_0^t B(t-a)\, p(a)\, n(a)\, m(a)\, da \tag{18}$$

the same born to the immigrants. The contribution of the immigrants to the growth of population may be measured by comparing $B_i(t)$ and $B_o(t)$.

The estimate of the second generation immigrants, is an important component of the population size. This is so since in many respects the life experiences of children (especially those who are young), who accompany their immigrant parents are not that different from those that are born to such parents in the host country. Given $B_i(t)$, the size of this second generation immigrants can be obtained from

$$I_2(t) = \int_0^t B_i(t-u)\, p(u)\, du \tag{19}$$

Thus, insofar as the impact of immigration is dependent on the relative size of the population that is culturally different from the host population it seems that size of the former should better be estimated by $I(t) + I_2(t)$ rather than by $I(t)$ alone.

We would like to conclude this section by noting some of the simplifications that can be achieved by assuming indefinite continuation of the vital and the immigration rates. It is immediately apparent from (15) that the model will approach stability and the birth trajectory will, in the long run, be subjected to a growth rate r, given by the real root of the integral equation

$$\int_0^\infty e^{-ra}\, t(a)\, m(a)\, da = 1 \tag{20}$$

That is to say, as $t \rightarrow \infty$ and a is finite

$$B(t-a) = B(t)\, e^{-ra} \tag{21}$$

The substitution of this long term solution in the functions defined earlier will result in interesting simplifications. For example, the limiting value of $i(t)$ in (14) will become

$$i = \frac{\int_0^\infty e^{-ra}\, p(a)\, n(a)\, da}{\int_0^\infty e^{-ra}\, p(a)\, da} \tag{22}$$

A similar expression can be written for the ratio of $B_i(t)$ and $B_e(t)$ from (18) and (17) to obtain the ratio of births to foreign born and native mothers respectively.

Once again, we shall note that the results derived in this section are based on an unchanging schedule of age-specific immigration rates and all the solutions have the same general form for all values of r (positive, zero or negative) or for all values of the net reproduction rate

$$R = \int_0^{P} p(a)\, m(a)\, da \tag{23}$$

The overall contribution of immigration can also be examined by comparing the intrinsic rates of growth with and without the immigration component. This means comparing the solution of (20) with that of a similar equation in which $t(a)$ is replaced by $p(a)$.

As noted in the first section, we shall now turn our attention to the special case in which $R < 1$ and the size as well as age composition of the immigrants remain the same every year.

3. Constant Number of Immigrants by Age and Below Replacement Level Net Reproduction Rate

If $J(a)$ is the number of immigrants aged a at any time then the number of children born to them at time t assuming as before that the immigrants adopt the net maternity rate of the host population, is given by (Espenshade, *ibid)*

$$B_I(t) = \int_0^{\infty} \int_0^{t} J(a-s)\, \frac{P(a)}{P(a-s)}\, m(a)\, ds\, da \tag{24}$$

Observe that the size of the immigrant population aged 0 at time t is given by

$$I(a,t) = \int_0^{t} J(a-s)\, \frac{p(a)}{p(a-s)}\, ds \tag{25}$$

which for large t becomes independent of t i.e., $I(a)$ for all ages, so that we can write

$$B_I(t) = \int_0^{\infty} I(a)\, m(a)\, da = B_I \tag{26}$$

It has been shown earlier that the limiting value of the total number of births $B(t)$ to such a population is given by (Espenshadc, *ibid;* Mitra, 1985)

$$B(t) = \frac{B_I}{1-R} = B \tag{27}$$

which is a constant. Subtracting B_I from (27), we get the number of babies born to the native population as

$$B_c = B_I \frac{R}{1 - R} \tag{28}$$

If

$$I = \int_0^\infty I(a)\, da \tag{29}$$

stands for the size of the immigrant population, then the native born will equal

$$C = \int_0^\infty Bp(a)\, da = Be\ (0) \tag{30}$$

where $e(0)$ stands for the expectation of life. Similarly, the size of the second generation immigrants will be given by

$$I_2 = \int_0^\infty B_I p(a)\, da = B_I e(0) \tag{31}$$

The rates and ratios defined in the previous section can be easily obtained from the foregoing results. For example, the age-specific immigration rate $i(a)$ can be expressed as a ratio of the number aged a immigrating at any given time or $J(a)$ to the population aged a, namely $Bp(a) + I(a)$. That is to say,

$$i(a) = \frac{J(a)}{B_p(a) + I(a)} \tag{32}$$

The overall immigration rate i can be obtained by first summing the numerator and the denominator of (32) separately over age and then computing the ratio as

$$i = \frac{J}{Be(0) + I} \tag{33}$$

The other relevant characteristics of the population can be similarly obtained.

4. The Proportions of First and Second Generation Immigrants

A. Constant Age-Specific Immigration Rates

Observe from (12) and (13) that the total population is given by $I(t) + C(t)$ and therefore the proportion of immigrants to the total population is given by

$$p_t(i, 1) = \frac{I(t)}{I(t) + C(t)} = \frac{i(t)}{i(t) + 1} \tag{34}$$

from (14). The limiting value of (34) is (see 22)

$$p(i, 1) = \frac{i}{i+1} = \frac{\int_0^\infty e^{-ra}\, p(d)\, n(a)\, da}{\int_0^\infty e^{-ra}\, p(a)\, [1 + n(a)]\, da} \qquad (35)$$

Similarly, the proportion of second generation immigrants to the total population is given by

$$p_t(i, 2) = \frac{I_2(t)}{I(t) + C(t)} \qquad (36)$$

Note that $I_2(t)$ does not appear separately in the denominator since being native born it is included in $C(t)$. The expression (19) of $I_2(t)$ can also be written as

$$I_2(t) = k \int_0^\infty B(t - a)\, p(a)\, da \qquad (37)$$

where

$$k = \frac{B_i(t)}{B(t)} \qquad (38)$$

is the limiting value of the ratio of births to immigrants to the total births. Thus the limiting value of (36) can be expressed as

$$p(i, 2) = \frac{k \int_0^\infty e^{-ra} p(a)\, da}{\int_0^\infty e^{-ra}\, p(a)\, [1 + n(a)]\, da} \qquad (39)$$

after some simplification. Combining (35) and (39) one can write

$$p(i, 1) + \frac{p(i, 2)}{k} = 1 \qquad (40)$$

or

$$p(i, 2) = k[1 - p(i, 1)] \qquad (41)$$

Thus given $p(i, 1) = .02$ which is to say that 2 percent of the population are first generation immigrants and $k = .03$ or three percent of the births are to immigrants, the second generation immigrants works out as almost 3 percent of the total population. It may be noted that like $p(i, 1)$, k is also determined by $n(a)$ in addition to the other parameters.

B. Constant Number of Immigrants by Age and R < 1

From the equations (29) and (30) we can express the limiting value of the ratio of the immigrant to the total population as

$$p^*(i, 1) = \frac{I}{I + C} = \frac{I}{I + Be(0)} \tag{42}$$

and from (27) and (31), we get the same for the second generation immigrants as

$$p^*(i, 2) = \frac{I_2}{I + C} = \frac{B(1 - R)\, e(0)}{I + Be(0)} \tag{43}$$

Expressing I as

$$I = \frac{p^*(i, 1)}{1 - p^*(i, 1)}\, Be\,(0) \tag{44}$$

from (42) and substituting (44) in (43) we get

$$p^*(i, 2) = [1 - p^*\,(i, 1)]\,(1 - R) \tag{45}$$

So given $R = .95$ and $p^*(i, 1) = .02$, we get $p^*(i, 2) = .049$ or about 5 percent of the total population. Formulas like these are quite helpful in estimating future population compositions under conditions specified in the model.

5. Summary

The effects of several patterns of immigration on age structure and the birth trajectory have been investigated in this paper. For reasons of operational convenience, the mortality and fertility patterns have been assumed constant and in addition to that the immigrants have also been assumed to adopt the vital rates of their host country immediately upon their arrival. Two specific patterns of immigration have been looked into. In the first, the age-specific immigration rates, like the vital rates, remain invariant over time. In the second, a constant number of immigrants with unchanging age composition enter every year. In the first example, the contribution of the immigration component may be determined by comparing the difference it makes in the intrinsic rates of growth of the trajectory of births. In addition, the overall immigration rate may also be computed to examine the same effect. It may be noted that the intrinsic rates of birth and death will also be affected by immigration even when the age-specific rates remain the same. In the second example, the long term effect of a constant stream of immigration on the host population turns out to be significant only when the net reproduction rate is below replacement level. In both of these examples, interesting simplifications have been achieved in deriving some of the characteristics of the population composition which include, among others, the size and proportion of the second generation immigrants.

References

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